A METHOD FOR ESTIMATING THE TECHNICO-ECONOMIC EQUIVALENT STATES IN POWER SYSTEM OPERATION

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ABSTRACT

This paper mentions a method to appreciate the optimum conditions of p_power generation between electric plants in a power system using diagonalization of an estimative matrix relating to the p_power loss coefficients and the system economic incremental fuel cost. The largest eigenvalue of the estimative matrix allows us to determine a set of parameters of power system operation which are considered as technico-economic equivalencies; therefore, the economic fuel cost increment limit can be estimated because of complex change of power system operation in reality.

1. INTRODUCTION

One of the characteristics which is related to optimization of power system operation condition is the non-linearity of the system fuel cost function in the vicinity of optimum operation point. In addition, the parameters of initial state of power system such as the generator fuel cost functions, the bus load powers, the network resistances, inductances and capacitances, etc. are approximately determined in some probability in the reality. Finally, the optimization algorithm with some simplified assumtion can result in small errors. The combination of the above issues will cause not only one optimum operation point but also an technico-economic equivalent area where exist many technico-economic equivalent operation points. To compute the size of the technico-economic equivalent area relating to the deviations of optimum p_power generation is a complex problem. If the size of the technico-economic equivalent area is found, then we can estimate the accuracy of the realization of optimum condition in power system operation.

2. ESTIMATIVE MATRIX

Let's examine an electrical power system comprising n heat-power plants. In all practical cases, the fuel cost of generator i can be represented as quadratic function C_i of real power generation P_{ig}

(1) $$C_i = a_i P^2_{ig} + b_i P_{ig} + c_i \,;\quad i=1,2,\ldots,n.$$

Let's denote P_{iog} is the optimum p_power output of generator i, the optimum incremental fuel cost IC_{io} can be written as (2)

(2) $$IC_{io} = 2a_i P_{iog} + b_i \,;\quad i=2,3,...,n.$$

If the p_power output changes as $P_{ig}=P_{iog}+\Delta P_i$ then the fuel cost increment will change as (3)

(3) $$\Delta C_i = a_i \Delta P^2_i + IC_{io}\Delta P_i \,;$$

and the system loss increment will be (4)

(4) $$d\Delta P = 2\sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij} P_{jog}\Delta P_i + \sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}\Delta P_i \Delta P_j \,;$$

where B_{ij} denotes the elements of a loss coefficient matrix which can be found from [1]

$$B_{ij} = \frac{\cos(\alpha_i - \alpha_j)}{V_i V_j \cos\varphi_i \cos\varphi_j} \times \sum_{k=1}^{Nb} \frac{\left|\sum_{m=1}^{N} C_{km} J_m - C_{ki} J_\Sigma\right| \times \left|\sum_{m=1}^{N} C_{km} J_m - C_{kj} J_\Sigma\right|}{\left|J_\Sigma\right|^2} \,;$$

where n is the generator bus quantity; N is the load bus quantity; Nb is the branch quantity; C_{km} , C_{ki} , C_{kj} are the elements of current distribution factor matrix; J_m is the load current at bus m; J_Σ is the sum of generator currents; V_i, V_j are the generator voltages; α_i, α_j are the angles between current and voltage of generators i, j; $\cos\varphi_i$, $\cos\varphi_j$ are the power factors of generators i, j.

In general, the generator bus quantity n is less than the bus total quantity in the whole power system. The loss coefficient matrix B must be a square matrix of (n×n) size, then

(5) $$d\Delta P = \sum_{i=1}^{n} \frac{\partial \Delta P}{\partial P_{ig}} \Delta P_i + \sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}\Delta P_i \Delta P_j ;$$

The system loss increment $d\Delta P$ must be equal to the change of p_power outputs of all generators running in the power system, thus

(6) $$\sum_{i=1}^{n} \Delta P_i = \sum_{i=1}^{n} \frac{\partial \Delta P}{\partial P_{ig}} \Delta P_i + \sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}\Delta P_i \Delta P_j ;$$

(7) $$\sum_{i=1}^{n} \left(1 - \frac{\partial \Delta P}{\partial P_{ig}}\right) \Delta P_i = \sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}\Delta P_i \Delta P_j ;$$

The system fuel cost increment is

(8) $$\Delta C = \sum_{i=1}^{n} \left(IC_{io} \Delta P_i + a_i \Delta P_i^2 \right);$$

The optimum condition will be obtained when

(9) $$\lambda \left(1 - \frac{\partial \Delta P}{\partial P_{ig}}\right) = IC_{io} ;$$

where λ denotes the system incremental fuel cost. After some transfomation of (9) and then associating with (7)-(8) we obtain

(10) $$\Delta C = \lambda \sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}\Delta P_i \Delta P_j + \sum_{i=1}^{n} a_i \Delta P_i^2 ;$$

Therefore, the system fuel cost increment ΔC is determined as a quadratic function of the generator p_power output deviations ΔP_i with an estimative matrix E

$$E = \begin{bmatrix} \lambda B_{11} + a_1 & \lambda B_{12} & \dots & \lambda B_{1n} \\ \lambda B_{21} & \lambda B_{22} + a_2 & \dots & \lambda B_{2n} \\ \dots & \dots & \dots & \dots \\ \lambda B_{n1} & \lambda B_{n2} & \dots & \lambda B_{nn} + a_n \end{bmatrix};$$

3. TECHNICO-ECONOMIC EQUIVALENT STATES

Observing a space of n dimensions corresponding to ΔP_i (i=1,2,..,n) and the (n+1) dimension corresponding to the system fuel increment ΔC, a globoil space exists around the optimum point with a radius R.

The R magnitude determines the proximity state level to the optimum point, then

(11) $$R = \sqrt{\sum_{i=1}^{n} \Delta P_i^2} ;$$

Assume that the existing ΔP_i (i=1,2,…,n) satisfy to an inequality

(12) $$\sum_{i=1}^{n} \Delta P_i^2 \le R^2 ;$$

then

(13) $$\Delta C_{max} \le HR^2 ;$$

where H is a number which must be found to appreciate the sytem fuel cost increment. If a maximum eigenvalue ε_{max} of the estimative matrix is computed, then it will be accepted [2] as the number H in (13); in this case we have

(14) $$\Delta C \le \varepsilon_{max} \sum_{i=1}^{n} \Delta P_i^2 ;$$

4. EXAMPLE

Let's investigate a 220kV power system consisting of 3 generators and of 5 loads. The system's schema is shown in figure 1

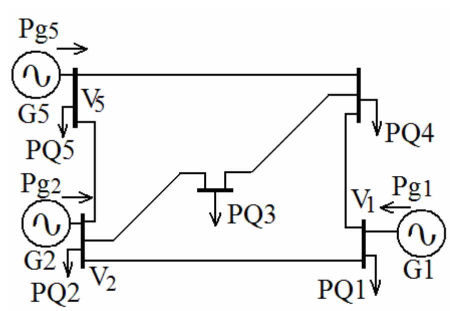

Fig.1-Schema of Power System

The linedata is given in table 1

Tab.1 -Branch RXB

bus	bus	R(Ω)	X(Ω)	0.5B (Ω^{-1})
1	2	15	46	1.55E-04
1	4	12	38	1.26E-04
2	3	10	30	1.00E-04
2	5	13	39	1.31E-04
3	4	11	34	1.13E-04
4	5	16	48	1.60E-04

The numerical results of initial state parameters calculation are presented in table 2

Tab.2 - Initial state parameters

	Voltage	Generator		Load	
bus	pu	MW	MVAr	MW	MVAr
1	1.0197	160	30	55	15
2	1.0151	175	35	95	35
3	0.9654	0	0	175	40
4	0.9781	0	0	195	45
5	1.0455	284.46	61.935	85	25
System fuel cost C=3781.021$/h P_power loss ΔP=14.455MW					

The generator fuel cost functions are given as follow

$C_1=0.007P^2_{g1}+3.9P_{g1}+155$, \$/h; $(50\le P_{g1}\le 275)$MW;
$C_2=0.006P^2_{g2}+4.5P_{g2}+149$, \$/h; $(50\le P_{g2}\le 250)$MW;
$C_5=0.008P^2_{g5}+3.2P_{g5}+145$, \$/h; $(50\le P_{g5}\le 300)$MW;

We can carry out the power system steady-state optimization [1] as shown in figure 2

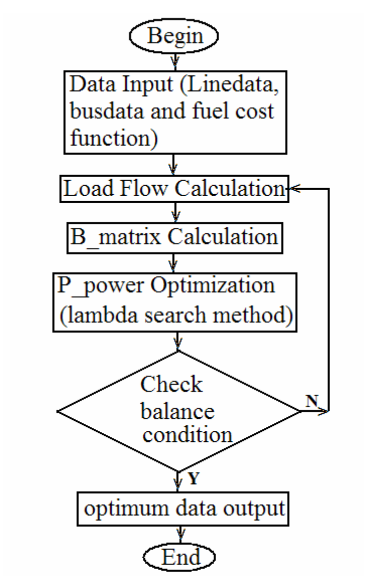

Fig.2 - State optimization flow-chart

The main numerical results of the problem of economic p_power geneation are presented in table 3

Tab.3 - Economic state parameters

	Voltage	Generator		Load	
bus	pu	MW	MVAr	MW	MVAr
1	1.0346	204.20	30	55	15
2	1.0254	194.84	35	95	35
3	0.9759	0	0	175	40
4	0.9880	0	0	195	45
5	1.0455	218.82	55.989	85	25
System fuel cost C=3725.113 $/h P_power loss ΔP=12.862MW					

The loss coefficient matrix is

$$B=\begin{bmatrix}0.915773 & 0.118120 & 0.006307\\ 0.118120 & 0.490668 & 0.162591\\ 0.006307 & 0.162591 & 1.002288\end{bmatrix}\times 10^{-4};$$

The system incremental fuel cost is $\lambda=7.05723$;

In this case the estimative matrix E is

$$E=\begin{bmatrix}0.76463 & 0.00834 & 0.00045\\ 0.00834 & 0.63463 & 0.01147\\ 0.00045 & 0.01147 & 0.87073\end{bmatrix}\times 10^{-2};$$

The modal matrix of the estimative matrix is

$$M_E=\begin{bmatrix}-0.0632 & 0.9980 & 0.0080\\ 0.9968 & 0.0628 & 0.0487\\ -0.0481 & -0.0110 & 0.9988\end{bmatrix};$$

The eigenvalues are

$$\varepsilon=\begin{bmatrix}0.0063 & 0 & 0\\ 0 & 0.0077 & 0\\ 0 & 0 & 0.0087\end{bmatrix};$$

And we have $H = 0.0087$;

If the maximum system fuel cost increment is accepted in range of 1.5\$/h, then the technico-economic equivalent states must be found in a globoil space (precision of economic equivalent state) with radius R= 13.13

4.1. Case study 1.

Assume that the generator p_power outputs deviating from the optimum p_power magnitudes as ΔP_1=4.2MW; ΔP_2=4.84MW; and ΔP_3=9.19MW.

Estimating fuel cost increment from (14), we have ΔC=1.092$/h (<1.5$/h). Some more results in detail are presented in table 4

Tab.4 - Equivalent state parameters

	Voltage	Generator		Load	
bus	pu	MW	MVAr	MW	MVAr
1	1.0328	200	30	55	15
2	1.0239	190	35	95	35
3	0.9745	0	0	175	40
4	0.9868	0	0	195	45
5	1.0455	228.01	56.56	85	25
System fuel cost C= 3726.081 $/h P_power loss ΔP=13.00MW					

4.2. Case study 2.

Assum that the generator G1 voltage V_1 is adjusted so that its power output changes into 204.2+j50MVA, and the generator G2 voltage V_2 is adjusted so that its power output changes into 194.84+j45MVA. In this case, the generator G5 voltage V_5 is holded on 231kV so that its power output is 218.3+j21.618MVA. Estimating the fuel cost increment from (14), we have ΔC=0.0024$/h. This small value of fuel cost increment means that the generator voltage and q_power change will also result in an economic equivalent state. In this case, the economic equivalent state parameters are presented in table 5

Tab.5 Equivalent state parameter

	Voltage	Generator		Load	
bus	pu	MW	MVAr	MW	MVAr
1	1.0619	204.20	50	55	15
2	1.0450	194.84	45	95	35
3	0.9962	0	0	175	40
4	1.0072	0	0	195	45
5	1.05	218.30	21.618	85	25

5. CONCLUSION

Using the H and R magnitudes we can analyse the influence of power system parameters on the technico-economic equivalent state in electrical power system operation with an estimative matrix obtained from the results of a power system state optimization solving.

The general forms of loss coefficient expression given in [2]-[3]-[4], without some necessary transformations, are unsuitable for the formulas as introduced above. This paper develops a method estimating the technico-economic equivalent state with an estimative matrix which is determined by a square and symmetric loss coefficient matrix of (n×n) size, with n is the generator quantity in a power system, including the slack bus generator.

The algorithm solving this problem can be developed by using the system eigen-image vector in [5] to estimate the fuel cost increment without the diagonalization of the estimative matrix.

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